Reason-based Deontic Logic

Alessandro Giordani

Catholic University of Milan
Largo A. Gemelli, 1 - 20123 Milan, Italy
alessandro.giordani@unicatt.it

Abstract

This paper introduces a system of deontic logic based on the idea that obligations are grounded on reasons. A reason-based deontic system is worth considering for at least three reasons: it may shed light on the way in which obligations are generated; it allows us to cope with conflicts between reasons while avoiding conflicts between obligations; finally, it may help us to assess the question as to whether standard deontic logic is appropriate to model basic deontic reasoning. The system I propose is developed in a framework that combines standard and neighborhood semantics and it is proved to be sufficiently powerful to represent ordinary deontic reasoning and to successfully address some significant problems in deontic logic.

Keywords: practical reasons, *pro tanto* obligations, all things considered obligations.

1 Introduction

The aim of this paper is to develop a modal system of deontic logic based on the idea that consistent obligations are grounded on possibly inconsistent reasons. [1] This project, whose significance is due to the prominent role currently attributed to reasons in the study of normative concepts and normative systems, [2] has to address two general issues: (*i*) from the philosophical side, to devise what basic principles about reasons are to be assumed to deduce consistent oughts without incurring in counter-intuitive consequences; (*ii*) from the logical side, to construct a system of deontic logic characterized by those principles in a suitable semantic framework. These issues are taken into account in the following three sections. In section 2 the basic principles underlying the system are proposed, as emerging from the recent debate on the connections between reasons and obligations. In section 3, after having characterized the system both from a semantic and from an axiomatic point of view, it is shown that it can be exploited to solve some interesting deontic problems and that it

[1] The notions of reason and obligation we will cope with are the notions of *pro tanto* objective normative reason and *pro toto* objective normative obligation [7, ch.1 and ch.4].

[2] See [7], [19, part I], [20, ch.1], [21, ch.4], and [22, ch.4] for extensive and insightful presentations of the topic. See [25, especially part III and V] for an up-to-date discussion of the structure and the role played by reasons in practical argumentation and deliberation.

enables us to vindicate standard deontic logic as the logic of basic deontic reasoning. In the final section two recent accounts, similar in scope, are considered and compared with the present one, and it is shown that the basic intuitions on which those accounts rely can be appropriately interpreted in it.

2 Intuitive principles

We want to be able to capture basic forms of deontic reasoning involving connections between reasons and obligations. In this respect, two inference schemata are considered as highly desirable in the literature [11,16,17]: a schema for implicative reasoning (*IR*) and a schema for disjunctive reasoning (*DR*). These schemata can be shown to be valid if we assume two intuitive principles concerning why certain reasons follow from the presence of other reasons, namely *Consistent Closure* and *Consistent Conjunction*. However, the assumption of these principles generates a problem of deontic explosion. What we want is then a system of deontic logic enabling us both to derive *IR* and *DR*, given an appropriate interpretation of *Consistent Closure* and *Consistent Conjunction*, and to avoid undesired consequences. [3] The idea underlying the system here developed is that obligations are based on reasons and reasons are distinguished into three kinds: basic reasons, which are the central elements of deontic reasoning; combined reasons, which allow for a principle of *Consistent Conjunction*, but are not closed under consistent closure; and derivative reasons, which allow for a principle of *Consistent Closure*, but are not closed under consistent conjunction. As we will see, in such system, besides solving paradigmatic cases of deontic dilemmas, we can derive specific versions of *IR* and *DR* (theorem 3.15 below), while avoiding explosions (theorem 3.13 below).

Let us begin with presenting *IR* and *DR*. Let a be a generic agent.

(i) **Implicative reasoning** (*IR*):
  (a) it is obligatory for a to do ϕ;
  (b) ϕ entails ψ;
  (c) therefore, a has a reason to do ψ.

(ii) **Disjunctive reasoning** (*DR*):
  (a) it is obligatory to do $\phi \vee \psi$;
  (b) a has a reason to do $\neg\phi$;
  (c) therefore, a has a reason to do ψ.

Both kinds of reasoning are acceptable, provided we assume that obligations are based on reasons and that some intuitive closure principles concerning reasons are logically valid. Specifically, as to IR, suppose that obligations are based on reasons and that it is obligatory for a to do ϕ; then a has a reason to do ϕ; thus, *if* having a reason to do ϕ implies having a reason to do all that ϕ entails, *then* a has a reason to do ψ. Similarly, as to DR, suppose that obligations are based on reasons and that it is obligatory for a to do $\phi \vee \psi$; then a has a reason

[3] See [16] for an exposition of the current debate and an analysis of the analogies between cases of conflicting reasons and cases of conflicting obligations.

to do $\phi \vee \psi$; thus, *if* a has a reason to do $\neg\phi$ and having reasons to do two conjuncts implies having a reason to do the conjunction, *then* a has a reason to do $(\phi \vee \psi) \wedge \neg\phi$; hence, if having a reason to do something entails having a reason to do all that is entailed by that thing, a has a reason to do ψ. In sum, if we allow for principles like:

(*Closure*) if ψ is a necessary condition of ϕ, then having a reason to do ϕ implies having a reason to do ψ;

(*Conjunction*) having a reason to do ϕ and having a reason to do ψ implies having a reason to do $\phi \wedge \psi$;

then we are able to account for the validity of IR and DR. Besides, as we can check by analyzing the arguments just proposed, both schemata are derivable from the following conditioned versions of *Closure* and *Conjunction*:

(*Consistent Closure*) if ϕ is possible and ψ is a necessary condition of ϕ, then having a reason to do ϕ implies having a reason to do ψ;

(*Consistent Conjunction*) having a reason to do ϕ and having a reason to do ψ implies having a reason to do $\phi \wedge \psi$, if it is possible to do $\phi \wedge \psi$.

Hence, IR and DR turn out to hold under very mild assumptions.

2.1 Problems

When considering the consequences of adopting IR and DR, we encounter two basic problems [11]. The first and lighter one is that, under the intuitive assumption that reasons can conflict and that there is no reason for doing something impossible, *Conjunction* is untenable. To be sure, it is impossible to allow for conflicts of reasons, since having a reason for ϕ and a reason for $\neg\phi$ would immediately entail having a reason for $\phi \wedge \neg\phi$. Thus, *Conjunction* is to be abandoned. The second problem is more pressing. Suppose that we have a reason to do ϕ and a reason to do $\neg\phi$, and that both ϕ and $\neg\phi$ are possible. Suppose also that something, say doing ψ, does not entail doing ϕ, so that we can do ψ without doing ϕ. Since ϕ entails $\phi \vee \psi$, we have a reason to do $\phi \vee \psi$, by *Consistent Closure*. Since it is possible to do $\neg\phi \wedge \psi$, it is also possible to do $\neg\phi \wedge (\phi \vee \psi)$, by propositional logic. Hence, by *Consistent Conjunction*, we have a reason to do $\neg\phi \wedge (\phi \vee \psi)$. Still, $\neg\phi \wedge (\phi \vee \psi)$ entails ψ, and so, by *Consistent Closure* we have a reason to do ψ. Thus, assuming that reasons can conflict, these two principles allow us to derive the following

Principle of Explosion. If we have conflicting reasons, then we have reasons to do anything independent of the content of the conflict.

The key problem to be addressed, in a framework allowing for rules corresponding to IR and DR, is then the following

Problem of Explosion. If we have conflicting reasons, how to avoid that anything independent of the content of the conflict be supported by a reason.

2.2 Strategies of solution

In light of the current debate on conflicts in deontic logic and the logic of reasons, two main strategies can be pursued in order to solve this problem. [4] According to the first one, we can put into question the validity of *Consistent Closure* and adopt a more limited principle to the effect that, if ψ is a necessary condition of ϕ, then having a reason to do ϕ entails having a reason to do ψ provided that we have no reason to avoid to do ϕ. It is not difficult to see that limiting the application of *Consistent Closure* this way blocks the possibility of inferring that we have a reason to do $\phi \vee \psi$ if we have a reason to do ϕ since, in the case we have considered, we also have a reason to do $\neg\phi$. According to the second strategy, we can put into question the validity of *Consistent Conjunction* and limit the application of the principle to a certain class of reasons, typically basic reasons, thus blocking the possibility of inferring that we have a reason to do $\neg\phi \wedge (\phi \vee \psi)$ if we have a reason to do $\neg\phi$ and a reason to do $\phi \vee \psi$, given that having a reason to do $\phi \vee \psi$ derives from having a reason to do ϕ. While both strategies are effective in preventing the derivation of explosions, the first one can do that only at a high cost, due to the fact that it prevents us from deriving that we have a reason to do $\phi \vee \psi$ if we have a reason to do ϕ and a reason to do ψ. To be sure, when we have a reason to do ϕ and a reason to do ψ, we would like to accept that we also have a reason to do one of ϕ and ψ, even though ϕ and ψ cannot be done together. [5]

The system we are going to introduce is designed, among other things, to allow for rules like IR and DR and to provide a solution to the problem of explosion along the lines of the second strategy.

3 A system of reason-based deontic logic

In this section system RDL of reason-based deontic logic is introduced. Its language should be rich enough to describe different ways of operating with reasons. In particular, when arguing about what to do, we typically combine reasons and infer the existence of reasons from the presence of other reasons. Then, if we become aware that some reasons generate conflicts, we select the strongest ones, or the ones that seem to be the strongest in the circumstances, and combine them to identify a definite course of action. In addition, when assessing our actions, we discern things which are done for a reason and things done without reason. To take into account these distinctions, modal operators are introduced for saying that a reason to do something is basic ($\mathsf{R_B}$), obtained by aggregation ($\mathsf{R_C}$), by derivation ($\mathsf{R_D}$), or by aggregating selected reasons ($\mathsf{S_C}$). Furthermore, I introduce two modal operators for saying that something which is the case is supported by a reason (R) or by a selected reason (S). Finally, two deontic operators are considered, for obligations based on generic reasons ($\mathsf{O_R}$) and obligations based on selected reasons ($\mathsf{O_S}$).

[4] See [17] and [10,11,12] for comprehensive presentations.

[5] This point is cogently defended in [6,13,14,16,17].

Definition 3.1 The language $\mathcal{L}_{RDL}$ of RDL is based on a set $\{p_i\}_{i\in\mathbb{N}}$ of propositional variables and is defined according to the following rules.

$$\phi ::= p_i \mid \neg\phi \mid \phi \wedge \phi \mid \Box\phi \mid \mathsf{R_B}\phi \mid \mathsf{R_C}\phi \mid \mathsf{R_D}\phi \mid \mathsf{R}\phi \mid \mathsf{O_R}\phi \mid \mathsf{S}\phi \mid \mathsf{S_C}\phi \mid \mathsf{O_S}\phi$$

$\mathcal{L}_{RDL}$ is a powerful language. This notwithstanding it can be interpreted in a very intuitive fashion on the basis of appropriate modal frames. Let us first present the intended meaning of the modal formulas. $\Box\phi$ states that ϕ is an unavoidable state of affairs under the circumstances. $\mathsf{R_B}\phi$ states that there is a *basic reason* to do ϕ. The notion of basic reason is here introduced as a primitive notion. [6] $\mathsf{R_C}\phi$ states that there is a *combined reason* to do ϕ, i.e., that ϕ is supported by a set of basic reasons opportunely combined, while $\mathsf{R_D}\phi$ states that there is a *derivative reason* to do ϕ, i.e., that ϕ is a consequence of something that is supported by a set of basic reasons opportunely combined. $\mathsf{R}\phi$ states that ϕ is the case in accordance with a reason, i.e., in typical cases, that the agent has seen to it that ϕ based on a certain reason. $\mathsf{S_C}\phi$ states that there is a *strong combined reason* to do ϕ, viewed as a reason that has passed a process of deliberation and selection run by the agent under specific circumstances, and $\mathsf{S}\phi$ states that ϕ is the case in accordance with a *strong reason*. Finally, a formula like $\mathsf{O_R}\phi$ states that ϕ is obligatory given the set of available reasons, while $\mathsf{O_S}\phi$ states that ϕ is obligatory given the set of available strong reasons. I will refer to $\mathsf{O_R}\phi$ and $\mathsf{O_S}\phi$ as reason-based obligations.

Remark 3.2 *Intuitively, we can use* $\mathsf{R_B}$ $\mathsf{R_C}$*,* $\mathsf{R_D}$ *to model different kinds of* pro tanto *practical reason,* $\mathsf{O_R}$ *to model the notion of* pro toto *practical reason and* $\mathsf{O_S}$ *to model the notion of* pro toto *or* all things considered *obligation.*

3.1 Semantics

The semantics for RDL builds on suitable combinations of neighborhood and standard semantics recently proposed in epistemic logic [7] and incorporates both a distinction between non-derivative and derivative reasons and a distinction between reasons and strong reasons. [8]

[6] This notion is widely used in epistemology, where a basic reason is associated with a basic source of justification, as acknowledged by standard foundationalist accounts. See [9] for an introduction and [1, ch.1 and ch.3] for further discussion. It is also becoming popular in ethics, where it parallels the notion of basic obligation: in some approaches, basic reasons are assumed to be primitive [7,16,21,22]; in others they are identified either with basic intrinsic desires and values [2] or with propositions constituting the antecedents of basic rules of action. In particular, the last interpretation is consistent both with the approach proposed in [21] and with the one developed in [13,14].

[7] Specifically, these semantics are used in evidence-based epistemic logic [26,27,28] and topological epistemic logic [3,4,5]. See [8,18] for an introduction to neighborhood semantics.

[8] The distinction between non-derivative and derivative reasons is the key element that will allow us to separate *Closure*, which is valid with respect to derivative reasons, from *Consistent Conjunction*, which is valid with respect to non-derivative combined reasons. A strategy based on this distinction is pursued in [16] to address the problem of explosion relative to reasons. The distinction between reasons and strong reasons is a version of the distinction between defeasible and defeasible but undefeated reasons [12,13,14].

Definition 3.3 A frame for $\mathcal{L}_{RDL}$ is a tuple $(W, \mathcal{R}, \mathcal{R}^+, \mathcal{S}^+)$, where $\mathcal{R}, \mathcal{R}^+, \mathcal{S}^+ \subseteq \wp(W)$ and W, $\mathcal{R}$, $\mathcal{R}^+$, $\mathcal{S}^+$ satisfy the following conditions

1. $\varnothing \neq W$;
2. $W \in \mathcal{R}$;
3. $\mathcal{R} \subseteq \mathcal{R}^+$;
4. if $X \in \mathcal{R}^+$, then $X \neq \varnothing$;
5. if $X \in \mathcal{R}^+$ and $Y \in \mathcal{R}^+$ and $X \cap Y \neq \varnothing$, then $X \cap Y \in \mathcal{R}^+$;
6. if $r(P) \neq \varnothing$, then $r(P) \in \mathcal{R}^+$, where $r(P) = \bigcup\{X \in \mathcal{R}^+ : X \subseteq P\}$;
7. if $X \in \mathcal{S}^+$ and $Y \in \mathcal{S}^+$ and $X \cap Y \neq \varnothing$, then $X \cap Y \in \mathcal{S}^+$;
8. if $s(P) \neq \varnothing$, then $s(P) \in \mathcal{S}^+$, where $s(P) = \bigcup\{X \in \mathcal{S}^+ : X \subseteq P\}$;
9. $W \in \mathcal{S}^+ \subseteq \mathcal{R}^+$.

In light of conditions 5, 6, we say that $\mathcal{R}^+$ is closed under *consistent aggregation* and *conditioned addition.* Let us comment on these elements in turn.

W is a set of states, viewed as the set of scenarios that are consistent with the background situation in which an agent is located, that is the set of scenarios that are possible given what is settled in the background. Condition 1 ensures that the background itself is consistent, so that there are indeed possible states.

$\mathcal{R}$ is a set of elements related to the basic reasons of an agent. In this framework $\mathcal{R}$ is sufficiently abstract to allow for different interpretations. In more detail, $\mathcal{R}$ can be interpreted in at least two different ways.

(i) As *a set of objectives identified with the agent's basic reasons.* The intuitive sense of $X \in \mathcal{R}$ is then that X is a basic reason viewed as an intrinsic value to be realized, so that X is the set of states where that value is actually realized. Accordingly, the interpretation of $X \subseteq P$ is that P is implied by one of the agent's basic reasons.

(ii) As *a set of propositions supported by the agent's basic reasons.* The intuitive sense of $X \in \mathcal{R}$ is then that there is a basic reason to do X, so that X is a proposition supported by the agent's basic reasons. Accordingly, the interpretation of $X \subseteq P$ is that P is *indirectly* supported by the agent's basic reasons, being entailed by X, which is *directly* supported by them.

Here I assume the second interpretation, under the general proviso that having a reason to do something, say P, is to be understood as having a reason (i) to do P, if P is not settled given the background and not realized, (ii) to preserve P, if P is not settled but realized, or (iii) to take P into account, if P is settled given the background. Hence, in light of (iii), condition 2, stating that W is in $\mathcal{R}$, captures the intuitive principle that an agent has always to take into account what is settled given the background.

$\mathcal{R}^+$ is the set of propositions supported by the *combined reasons* available to an agent, that is the set containing the propositions that an agent can support by combining basic reasons. Condition 3 states that $\mathcal{R}$ is a subset of $\mathcal{R}^+$, which corresponds to the requirement that taking a reason as it stands is a way of combining reasons. Condition 4 states that propositions supported by combined reasons, and so also by basic reasons, are consistent. The underlying idea is that an agent is able to combine reasons in a consistent way, and so that no combination of basic reasons supports a contradiction. Crucially, the fact that combined reasons are consistent in themselves does not exclude the possibility of conflicting reasons, that is of reasons supporting inconsistent propositions. To be sure, the fact that $X \in \mathcal{R}^+$ implies $X \neq \varnothing$ does not exclude the possibility that, for some $X, Y \in \mathcal{R}^+$, $X \cap Y = \varnothing$; what is excluded is only that $X \cap Y$ can be supported by a combined reasons, i.e., that $X \cap Y \in \mathcal{R}^+$. Finally, conditions 5 and 6 specify what kinds of operation of combination are available to an agent. Condition 5 underpins a principle of consistent conjunction. Indeed, this condition allows for operations of consistent aggregation, on the basis of which reasons that support two mutually consistent propositions are aggregated into a reason supporting their conjunction. Condition 6 allows for operations of conditioned addition, on the basis of which all the reasons supporting propositions that entail a proposition P can be added to obtain a new reason, which is in fact *the most stable reason* that supports P. Indeed, $r(P)$ is the union of all propositions that entail P and that are supported by some available reasons. Therefore, all the reasons supporting $r(P)$ provide support for propositions that are stronger than P, and so are less stable, being reasons that can be attacked with less difficulty.

Remark 3.4 *I will refer to $\mathcal{R}^+$ as the set of all reasons and to $\mathcal{S}^+$ as the set of all strong reasons, thus identifying the concepts of reason and strong reason with the concept of combined reason and combined strong reason.*

Finally, $\mathcal{S}^+$ is the set of propositions supported by the *strong combined reasons* available to an agent. Conditions 7 and 8 are analogous to the corresponding conditions on $\mathcal{R}^+$ and ensure that the operations of composition available to the agent are operative with respect to the reasons in $\mathcal{S}^+$ as well. Condition 9 states that $\mathcal{S}^+$ is a subset of $\mathcal{R}^+$, which follows from the definition of $\mathcal{S}^+$, and that W is in $\mathcal{S}^+$, which is intuitive given the characterization of the notion of reason. The idea behind the introduction of $\mathcal{S}^+$ is that, given a certain background and a certain set of initial reasons, and given the possibility of conflicts, an agent has to weigh up the reasons that are stronger under the circumstances and arrive at a decision based on them. In this respect, note that the present framework is not committed to a specific procedure for weighing reasons, since what is important for our purposes is just the outcome of the process, that is the set of reasons that are eventually selected.

Definition 3.5 A model for $\mathcal{L}_{RDL}$ is a tuple $M = (W, \mathcal{R}, \mathcal{R}^+, \mathcal{S}^+, V)$, where $(W, \mathcal{R}, \mathcal{R}^+, \mathcal{S}^+)$ is a frame for $\mathcal{L}_{RDL}$ and $V : \{p_i\}_{i \in \mathbb{N}} \to \wp(W)$ is a modal valuation assigning propositions to propositional variables.

The notion of truth is defined as follows.

Definition 3.6 Let $M = (W, \mathcal{R}, \mathcal{R}^+, \mathcal{S}^+, V)$ be a model for $\mathcal{L}_{RDL}$. The truth of ϕ at a world $w \in W$ in M is defined through the following conditions, where $[\phi]^M = \{w : M, w \models \phi\}$.

$M, w \models p_i \Leftrightarrow w \in V(p_i)$
$M, w \models \neg\phi \Leftrightarrow M, w \not\models \phi$
$M, w \models \phi \wedge \psi \Leftrightarrow M, w \models \phi$ and $M, w \models \psi$
$M, w \models \Box\phi \Leftrightarrow [\phi]^M = W$
$M, w \models \mathsf{R_B}\phi \Leftrightarrow [\phi]^M \in \mathcal{R}$
$M, w \models \mathsf{R_C}\phi \Leftrightarrow [\phi]^M \in \mathcal{R}^+$
$M, w \models \mathsf{R_D}\phi \Leftrightarrow \exists X \in \mathcal{R}^+(X \subseteq [\phi]^M)$
$M, w \models \mathsf{R}\phi \Leftrightarrow w \in r([\phi]^M)$
$M, w \models \mathsf{O_R}\phi \Leftrightarrow \forall X \in \mathcal{R}^+ \exists Y \in \mathcal{R}^+(Y \subseteq X \cap [\phi]^M)$
$M, w \models \mathsf{S_C}\phi \Leftrightarrow [\phi]^M \in \mathcal{S}^+$
$M, w \models \mathsf{S}\phi \Leftrightarrow w \in s([\phi]^M)$
$M, w \models \mathsf{O_S}\phi \Leftrightarrow \forall X \in \mathcal{S}^+ \exists Y \in \mathcal{S}^+(Y \subseteq X \cap [\phi]^M)$

The notion of logical consequence is defined as usual. So, $\Delta \Vdash_{RDL} \phi$ iff $M, w \models \Delta$ entails $M, w \models \phi$ for all $w \in W$ and models M for $\mathcal{L}_{RDL}$.

Definition 3.7 *RDL* is the logic of the class of models for $\mathcal{L}_{RDL}$.

The truth conditions reflect the intended meaning of the modal formulas. As expected, $\Box\phi$ is true just in case ϕ is true at all the states in W, $\mathsf{R_B}\phi$ is true just in case ϕ is supported by a basic reason in $\mathcal{R}$, and $\mathsf{R_C}\phi$ is true just in case ϕ is supported by a combined reason in $\mathcal{R}^+$. As to $\mathsf{R_D}$, the definition clarifies the distinction between non-derivative and derivative reasons in terms of the distinction between directly and indirectly supported propositions. [9] Thus, $\mathsf{R_D}\phi$ is true iff ϕ is supported by a derivative reason, i.e., iff ϕ is entailed by a proposition supported by a non-derivative reason in $\mathcal{R}^+$, i.e., iff ϕ is indirectly supported by a non-derivative reason in $\mathcal{R}^+$. $\mathsf{R}\phi$ is true just in case ϕ is true in a state in which what is supported by the most stable reason for ϕ is realized. Therefore, ϕ is the case and that ϕ is the case is in accordance with a reason, since ϕ is supported by its most stable reason. Similarly, $\mathsf{S}\phi$ is true just in case ϕ is true in a state in which what is supported by the most stable strong reason for ϕ is realized, while $\mathsf{S_C}\phi$ is true just in case ϕ is supported by a strong combined reason in $\mathcal{S}^+$. Lastly, $\mathsf{O_R}\phi$ is true just in case every reason in $\mathcal{R}^+$ can be strengthened to a reason for ϕ. Hence, being obligatory given the whole set of available reasons is interpreted as being supported in a set of reasons that do not conflict on what is obligatory. [10] Similarly, $\mathsf{O_S}\phi$ is true just in case every reason in $\mathcal{S}^+$ can be strengthened to a strong reason for ϕ.

It is worth noting that *RDL* models are generalizations of uniform models in standard deontic logic, that is models of the form $(W, Ideal)$. Indeed, let $M = (W, \mathcal{R}, \mathcal{R}^+, \mathcal{S}^+, V)$ be such that $\mathcal{R} = \mathcal{R}^+ = \mathcal{S}^+ = \{W, Ideal\}$, where $\varnothing \neq Ideal \subseteq W$. Then $Ideal \subseteq [\phi]^M$ iff $\forall X \in \mathcal{S}^+ \exists Y \in \mathcal{S}^+(Y \subseteq X \cap [\phi]^M)$.

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[9] In accordance with this definition, both basic and combined reasons count as non-derivative reasons, since both provide direct support to a proposition.

[10] It is not difficult to see that $\forall X \in \mathcal{R}^+ \exists Y \in \mathcal{R}^+(Y \subseteq X \cap [\phi]^M)$ if and only if $\forall X \in \mathcal{R}^+ \exists Y \in \mathcal{R}^+(Y \cap X \neq \varnothing$ and $Y \subseteq [\phi]^M)$. Thus, every reason in $\mathcal{R}$ is consistent with a reason for ϕ, and so ϕ is supported by reasons that are not in conflict relative to ϕ itself.

Thus, uniform standard deontic logic can be viewed as the logic determined by the class of models like M, where the standard notion of obligation is captured by O_S or, equivalently, by O_R. The underlying assumption in this case is that there is only one normative reason to be considered, namely the reason that is encoded in a consistent deontic code.

3.2 All things considered obligations

A standard approach for generating all things considered obligations from a set of available reasons has it that ϕ is obligatory when there is a *good* reason to perform ϕ, where the notion of good reason is defined in terms of an ordering relation on the set of reasons. [11] As an alternative, in line with the approach developed in [3,4,5,26,27,28], we may assume that it is obligatory to do ϕ when every reason can be strengthened to a reason for ϕ, that is when every reason is part of a set of reasons supporting ϕ. If every available reason can be strengthened to a reason for ϕ, then no available reason for ϕ can be outweighed by a stronger reason, and so the second approach is stricter than the first and generates less obligations. The approach I propose here is a combination of the ones just sketched and can be divided into two ideal stages. Suppose an agent is confronted with a deontic problem, e.g. whether she should do p. In the first stage, the agent implements the first approach by selecting within the set $\mathcal{R}^+$ of available reasons the set $\mathcal{S}^+$ of strong reasons, to be identified with the good reasons obtained after a process of deliberation. In the second stage, she checks whether every reason in $\mathcal{S}^+$ can be strengthened to a reason for p. If so, then she concludes that p is all things considered obligatory. Here, $\mathcal{S}^+$ can be thought of as generated from $\mathcal{R}^+$ by virtue of a suitable choice function, along the lines originally proposed in [23,24]. The reason why this procedure is adopted, instead of introducing an ordering relation on $\mathcal{R}^+$, is that it provides us with a more flexible device for modeling the outcome of a process of deliberation. In a more general setting, this approach can be developed in such a way that a set of triggered reasons $\mathcal{T}^+_w = \tau(w) \subseteq \mathcal{R}^+$ is assigned to each state w by a specific function τ, and then a set of strong reasons $\mathcal{S}^+_w = \sigma(w) \subseteq \mathcal{T}^+_w$ is selected by σ. In such a setting, various σs can be defined based on different properties of a choice function and the connections between the corresponding notions of all things considered obligation can be explored.

3.3 Axiomatization

Let us consider the following groups of axioms and rules.

Group 1: $KT5$ axioms and rules for $\Box$.

Group 2: KT axioms and rules for R and S.

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[11] This approach can be implemented in different ways. A common option is to assume that there is a good reason to perform ϕ iff there is an *undefeated* reason for ϕ, i.e., iff there is a reason for ϕ and there is no stronger reason, given the ordering, against the performance of ϕ. A more sophisticated option, which incorporates the notion of undefeated reasons, is put forward in [13,14]. Section 4.2 gives a hint of how this option can be handled in *RDL*.

Group 3: minimal axioms for $\mathsf{R_B}$, $\mathsf{R_C}$, $\mathsf{S_C}$, $\mathsf{R_D}$.

BN: $\Box(\phi \leftrightarrow \psi) \wedge \mathsf{R_B}\phi \rightarrow \mathsf{R_B}\psi$
RN: $\Box(\phi \leftrightarrow \psi) \wedge \mathsf{R_C}\phi \rightarrow \mathsf{R_C}\psi$
SN: $\Box(\phi \leftrightarrow \psi) \wedge \mathsf{S_C}\phi \rightarrow \mathsf{S_C}\psi$
DN: $\Box(\phi \rightarrow \psi) \wedge \mathsf{R_D}\phi \rightarrow \mathsf{R_D}\psi$

Consistent conjunction for $\mathsf{R_C}$ and $\mathsf{S_C}$:
RC: $\mathsf{R_C}\phi \wedge \mathsf{R_C}\psi \wedge \Diamond(\phi \wedge \psi) \rightarrow \mathsf{R_C}(\phi \wedge \psi)$
SC: $\mathsf{S_C}\phi \wedge \mathsf{S_C}\psi \wedge \Diamond(\phi \wedge \psi) \rightarrow \mathsf{S_C}(\phi \wedge \psi)$

Group 4: inclusions between modalities.

B1: $\mathsf{R_B}\phi \rightarrow \Box\mathsf{R_B}\phi$
B2: $\Box\phi \rightarrow \mathsf{R_B}\phi$

R1:	$\mathsf{R_C}\phi \rightarrow \Box\mathsf{R_C}\phi$	S1:	$\mathsf{S_C}\phi \rightarrow \Box\mathsf{S_C}\phi$	I1:	$\mathsf{S}\phi \rightarrow \mathsf{R}\phi$
R2:	$\Box\phi \rightarrow \mathsf{R_C}\phi$	S2:	$\Box\phi \rightarrow \mathsf{S_C}\phi$	I2:	$\mathsf{S_C}\phi \rightarrow \mathsf{R_C}\phi$
R3:	$\mathsf{R_C}\phi \wedge \phi \rightarrow \mathsf{R}\phi$	S3:	$\mathsf{S_C}\phi \wedge \phi \rightarrow \mathsf{S}\phi$	I3:	$\mathsf{R_B}\phi \rightarrow \mathsf{R_C}\phi$
R4:	$\Diamond\mathsf{R}\phi \rightarrow \mathsf{R_C}\mathsf{R}\phi$	S4:	$\Diamond\mathsf{S}\phi \rightarrow \mathsf{S_C}\mathsf{S}\phi$	I4:	$\mathsf{R_C}\phi \rightarrow \mathsf{R_D}\phi$
R5:	$\mathsf{O_R}\phi \leftrightarrow \Box\neg\mathsf{R}\neg\mathsf{R}\phi$	S5:	$\mathsf{O_S}\phi \leftrightarrow \Box\neg\mathsf{S}\neg\mathsf{S}\phi$	I5:	$\mathsf{R_D}\phi \leftrightarrow \Diamond\mathsf{R}\phi$

Theorem 3.8 *Axioms and rules in groups 1 – 5 are sound and complete with respect to the class of all models for $\mathcal{L}_{RDL}$.*

The proof is rather long and is presented in the extended version of the paper.

Fact 3.9 $\mathsf{O_R}\phi \rightarrow \mathsf{R_D}\phi$ *and* $\mathsf{O_S}\phi \rightarrow \mathsf{R_D}\phi$.

It follows from R5 and S5 by factivity of R and S, the logic of $\Box$, I5 and I1.

Fact 3.10 $\mathsf{R_C}$ *is not closed under necessary implication.*

To provide a counter-model for $\Box(\phi \rightarrow \psi) \wedge \mathsf{R_C}\phi \rightarrow \mathsf{R_C}\psi$ let M be such that $W = \{w_1, w_2, w_3\}$, $\mathcal{R} = \mathcal{R}^+ = \mathcal{S}^+ = \{W, \{w_1\}\}$, $V(p_1) = \{w_1\}$ and $V(p_2) = \{w_2\}$. Then, for all $w \in W$, $M, w \models \Box(p_1 \rightarrow p_1 \vee p_2)$, by the truth conditions of $\Box$, and $M, w \models \mathsf{R_C}p_1$, since $[p_1]^M = \{w_1\} \in \mathcal{R}^+$. Still, $M, w \models \mathsf{R_C}(p_1 \vee p_2)$ for no $w \in W$, since $\{w_1, w_2\} \notin \mathcal{R}^+$.

Fact 3.11 $\mathsf{R_D}$ *is not closed under consistent conjunction.*

To provide a counter-model for $\mathsf{R_D}\phi \wedge \mathsf{R_D}\psi \wedge \Diamond(\phi \wedge \psi) \rightarrow \mathsf{R_D}(\phi \wedge \psi)$ let M be such that $W = \{w_1, w_2, w_3\}$, $\mathcal{R} = \mathcal{R}^+ = \mathcal{S}^+ = \{W, \{w_1\}, \{w_2, w_3\}\}$, $V(p_1) = \{w_1\}$ and $V(p_2) = \{w_2\}$. Then, for all $w \in W$, $M, w \models \mathsf{R_C}(p_1 \vee p_2)$, since $[p_1]^M = \{w_1\} \in \mathcal{R}^+$ and $[p_1]^M \subseteq [p_1 \vee p_2]^M$, and $M, w \models \mathsf{R_D}\neg p_1$, since $[\neg p_1]^M = \{w_2, w_3\} \in \mathcal{R}^+$ and $M, w \models \Diamond((p_1 \vee p_2) \wedge \neg p_1)$, since $[(p_1 \vee p_2) \wedge \neg p_1]^M = \{w_2\}$. Still, $M, w \models \mathsf{R_D}((p_1 \vee p_2) \wedge \neg p_1)$, for no $w \in W$: $X \subseteq \{w_2\}$ for no $X \in \mathcal{R}^+$.

Fact 3.12 *Basic reasons do not entail obligations.*

To provide a counter-model for $\mathsf{R_B}\phi \rightarrow \mathsf{O_R}\phi$ let M be such that $W = \{w_1, w_2\}$, $\mathcal{R} = \mathcal{R}^+ = \mathcal{S}^+ = \{W, \{w_1\}, \{w_2\}\}$, $V(p_1) = \{w_1\}$. Then, for all $w \in W$, $M, w \models \mathsf{R_B}p_1$, since $[p_1]^M = \{w_1\} \in \mathcal{R}$. Still, $M, w \models \mathsf{O_R}p_1$ for no $w \in W$, since there is no $Y \in \mathcal{R}^+$ such that $Y \subseteq \{w_2\} \cap [\phi]$, given that $\{w_2\} \cap [\phi] = \varnothing$.

By I3 and I4 we obtain that neither derivative nor non-derivative reasons entail obligations. Furthermore, in accordance with the intuitive interpretation given in remark 3.2, we conclude that *pro tanto* obligations do not entail *pro toto* obligations, as it should be.

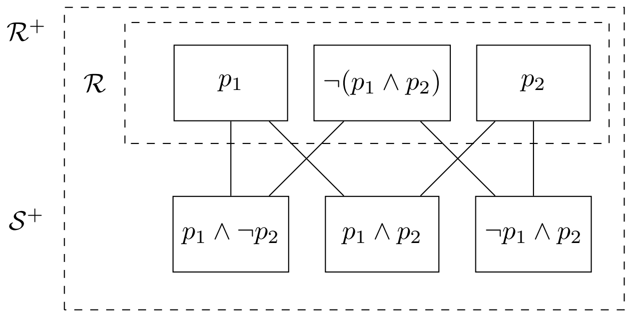
3.4 Solving a paradigmatic dilemma

In order to show how deontic reasoning is modeled in the present framework let us provide a solution to two versions of a paradigmatic dilemma.

> Alice promised both Bob and Carl that she would dine with them. The promises are equally important, but she prefer not to dine with them together, given that Bob doesn't like Carl.

In the first version Alice has no particular preference, while in the second version she would prefer to dine with Bob, since she is interested in him. Intuitively, we would like to derive that, in the first version, Alice ought to dine with one of them and that, in the second version, she ought to dine with Bob.

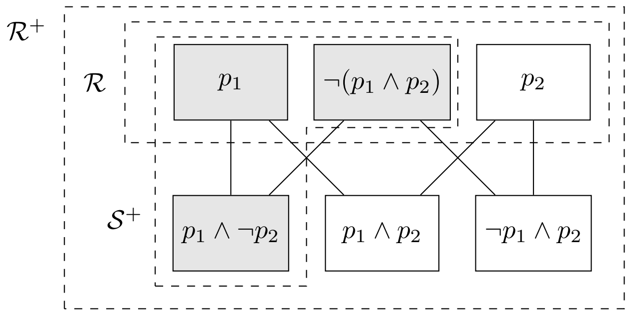
Model 1 $W = \{w_1, w_2, w_3\}$, $\mathcal{R} = \{W, \{w_1, w_2\}, \{w_1, w_3\}, \{w_2, w_3\}\}$, $\mathcal{R}^+ = \mathcal{S}^+ = \wp(W) - \{\varnothing\}$. Set $V(p_1) = \{w_1, w_2\}$ and $V(p_2) = \{w_2, w_3\}$, where p_1 stands for dining with Bob and p_2 stands for dining with Carl.



Model 1: only reasons different from W are represented

There is a reason for dining with Bob and a reason for dining with Carl, but there is no obligation for dining with one of them in particular, or with both, since the reasons in $\mathcal{R}^+$ cannot be strengthened to a reason for one of $p_1 \wedge \neg p_2$, $p_1 \wedge p_2$, $\neg p_1 \wedge p_2$. Still, there is an obligation to do $p_1 \vee p_2$, since any reason in $\mathcal{S}^+ = \mathcal{R}^+$ can be strengthened to a reason for doing $p_1 \vee p_2$.

Model 2 Let M be as before except that $\mathcal{S}^+ = \{W, \{w_1, w_2\}, \{w_1, w_3\}, \{w_1\}\}$.



Model 2: only reasons different from W are represented

Again, there is a reason for dining with Bob and a reason for dining with Carl, but now there is an obligation to dine with Bob, since any reason in $\mathcal{S}^+$ can be strengthened to a reason for doing $p_1 \wedge \neg p_2$, which is $\{w_1\}$.

3.5 Solving the problem of explosion

The present framework allows for a solution of the problem of explosion based on the distinction between non-derivative and derivative reasons. [12] In particular, in our system we can prove that there is no valid principle of explosion of the following form, where $C(\phi_1, \phi_2)$ stand for $\mathsf{R_B}\phi_1 \wedge \mathsf{R_B}\phi_2 \wedge \neg\Diamond(\phi_1 \wedge \phi_2)$ [13]

E1:	$\mathsf{R_B}\phi \wedge \mathsf{R_B}\neg\phi \rightarrow \mathsf{R_D}\psi$	GE1:	$C(\phi_1, \phi_2) \rightarrow \mathsf{R_D}\psi$
E2:	$\mathsf{R_B}\phi \wedge \mathsf{R_B}\neg\phi \wedge \Diamond\psi \rightarrow \mathsf{R_D}\psi$	GE2:	$C(\phi_1, \phi_2) \wedge \Diamond\psi \rightarrow \mathsf{R_D}\psi$
E3:	$\mathsf{R_B}\phi \wedge \mathsf{R_B}\neg\phi \wedge \neg\mathsf{R_D}\neg\psi \rightarrow \mathsf{R_D}\psi$	GE3:	$C(\phi_1, \phi_2) \wedge \neg\mathsf{R_D}\neg\psi \rightarrow \mathsf{R_D}\psi$

E2 entails E1, by propositional logic, and E3 entails E2, since $\Diamond\psi$ follows from $\neg\mathsf{R_D}\neg\psi$, by B2, I3, I4. In addition, the invalidity of the basic principles Ei entails the invalidity of the corresponding generalized principles GEi, and therefore it is sufficient to prove the following

Theorem 3.13 $\mathsf{R_D}\psi$ *is not a logical consequence of* $\{\mathsf{R_B}\phi, \mathsf{R_B}\neg\phi, \neg\mathsf{R_D}\neg\psi\}$. *Hence, basic reasons can conflict without implying that anything independent of the content of the conflict be supported by a reason.*

Let $W = \{w_1, w_2, w_3, w_4\}$, $\mathcal{R} = \mathcal{R}^+ = \mathcal{S}^+ = \{W, \{w_1, w_2\}, \{w_3, w_4\}\}$. Set $V(p_1) = \{w_1, w_2\}$ and $V(p_2) = \{w_1, w_3\}$, so that p_2 is independent of the content of p_1. Then, for all $w \in W$, $M, w \models \mathsf{R_B}p_1 \wedge \mathsf{R_B}\neg p_1$, by the definition of truth, and $M, w \models \neg\mathsf{R_D}\neg p_2$, since $X \cap [p_2]^M \neq \varnothing$ for all $X \subseteq \mathcal{R}^+$. Still, $M, w \models \mathsf{R_D}p_2$ for no $w \in W$, since $X \subseteq [p_2]^M$ for no $X \subseteq \mathcal{R}^+$.

Corollary 3.14 *None of* $\mathsf{R_B}\psi, \mathsf{R_C}\psi, \mathsf{R_D}\psi$ *follows from one of the sets obtained by substituting one of* $\{\mathsf{R_C}\psi, \mathsf{R_D}\psi\}$ *for* $\mathsf{R_B}\psi$ *in* $\{\mathsf{R_B}\phi, \mathsf{R_B}\neg\phi, \neg\mathsf{R_D}\neg\psi\}$ *or from one of the antecedents of GE1, GE2, GE3.*

Next, we show that RDL is powerful enough to derive IR and DR.

Theorem 3.15 *In RDL the rules corresponding to schemata* IR *and* DR *are derivable, when the reasons are interpreted as derivative reasons.*

Suppose $M, w \models \mathsf{O_R}\phi$ and $M, w \models \Box(\phi \rightarrow \psi)$. Then $\forall X \in \mathcal{R}^+ \exists Y \in \mathcal{R}^+(Y \subseteq X \cap [\phi]^M)$ and $[\phi]^M \subseteq [\psi]^M$, so that $\forall X \in \mathcal{R}^+ \exists Y \in \mathcal{R}^+(Y \subseteq X \cap [\psi]^M)$. Thus $\exists Y \in \mathcal{R}^+(Y \subseteq [\psi]^M)$; so $M, w \models \mathsf{R_D}\phi$. Hence $\mathsf{O_R}\phi, \Box(\phi \rightarrow \psi) \Vdash_{RDL} \mathsf{R_D}\psi$, and

$\mathsf{O_R}\phi, \Box(\phi \rightarrow \psi) \vdash_{RDL} \mathsf{R_D}\psi$, by theorem 3.8.

Suppose now $M, w \models \mathsf{O_R}(\phi \vee \psi)$ and $M, w \models \mathsf{R_C}\neg\phi$. Then $\forall X \in \mathcal{R}^+ \exists Y \in \mathcal{R}^+(Y \subseteq X \cap [\phi \vee \psi]^M)$ and $R \subseteq [\neg\phi]^M$ for some $R \in \mathcal{R}^+$. Thus $Y \subseteq R \cap [\phi \vee \psi]^M$

[12] See [16]. Since the present problem (involving conflicting reasons) has the same structure as the deontic problem of explosion (involving conflicting obligations), the solution in [16] is based on solutions for the deontic problem put forward e.g. in [14,15,29,30]. All these solutions share the idea of distinguishing two kinds of obligations.

[13] See [11] and [12, sec.5] for an in-depth discussion of the deontic versions of these principles.

for some $Y \in \mathcal{R}^+$; $Y \subseteq [\neg\phi]^M \cap [\phi \vee \psi]^M$ for some $Y \in \mathcal{R}^+$; $Y \subseteq [\psi]^M$ for some $Y \in \mathcal{R}^+$, and so $M, w \models \mathsf{R_D}\phi$. Hence $\mathsf{O_R}(\phi \vee \psi), \mathsf{R_B}\neg\phi \Vdash_{RDL} \mathsf{R_D}\psi$, so that

$\mathsf{O_R}(\phi \vee \psi), \mathsf{R_B}\neg\phi \vdash_{RDL} \mathsf{R_D}\psi$, by theorem 3.8.

A similar theorem can be proved when $\mathsf{O_S}$ is substituted for $\mathsf{O_R}$.

3.6 Logic of obligation

Finally, it can be proved that $\mathsf{O_R}$ and $\mathsf{O_S}$ are $KD45$ modalities, so that the logic of obligation is the system SDL of standard deontic logic.

Theorem 3.16 *The logic of* $\mathsf{O_R}$*, respectively* $\mathsf{O_S}$*, is* $KD45$*.*

It is sufficient to show that, for all sets $\Delta \cup \{\phi\}$ of formulas in the sublanguage of $\mathcal{L}_{RDL}$ containing $\mathsf{O_R}$, respectively $\mathsf{O_S}$, as the only modality, $\Delta \vdash_{KD45} \phi \Rightarrow \Delta \Vdash_{RDL} \phi \Rightarrow \Delta \Vdash_{KD45} \phi$, where $\Vdash_{KD45}$ is the relation of logical consequence based on the class of models $M = (W, R, V)$ in which $R : W \to \wp(W)$ is such that $v \in R(w) \Rightarrow R(v) = R(w)$. The proof is based on the fact that, as said above, uniform models for standard deontic logic can be viewed as specific RDL models, together with the fact that RDL models validates all $KD45$ axioms and rules. The full proof is included in the extended version of the paper.

As $\mathsf{O_R}$ and $\mathsf{O_S}$ are $KD45$ modality, obligations cannot conflict, while conflicts between reasons are allowed (theorem 3.13). This result is of interest inasmuch as it allows us to interpret SDL plus axioms 4 and 5 as the logic concerning consistent obligations selected on the basis of deliberation, and so to vindicate this extension of SDL as apt to model deontic reasoning about this kind of oughts. Note that, in the present context, axioms 4 and 5 are not problematic once properly understood. In fact, as per axioms $R2$ and $S2$, we have reasons to take into account what is settled given the background; but what is supported by a reason, and therefore also what is all things considered obligatory and permitted, is settled given the background. As a consequence, we have reasons to take into account our all things considered obligations and permissions, and this is what axioms 4 and 5 state.

4 Comparison with two related accounts

In this section I consider two versions of the systems put forward by McNamara and Horty to deal with conflicting obligations and reasons and show how they can be interpreted in the present framework. The choice of these systems is due to the fact that they inspired me in the construction of the framework.

4.1 McNamara's two-level system

This system accounts for the possibility of aggregating obligations in conflict-tolerant contexts. [14] The key idea is to introduce a distinction between *basic*, *derived* and *unproblematic derived* obligations, with corresponding operators

[14] See [15]. I will be only interested in the final part, which presents a distinction between different kinds of obligation in a minimal setting.

$\mathsf{O_0}$, $\mathsf{O_1}$, $\mathsf{O_U}$. In this framework a two-level model can be defined as a triple (W, Φ, V) where $W \neq \varnothing$, Φ is a finite set of formulas, and V is a modal valuation. To simplify the comparison, the truth conditions for modal formulas are given as follows, where $\subseteq_{Fin}$ is set-theoretical inclusion of a finite set. [15]

$M, w \models \Box\phi$ iff $[\phi]^M = W$;
$M, w \models \mathsf{O_0}\phi$ iff $\exists \phi_i \in \Phi([\phi]^M = [\phi_i]^M)$;
$M, w \models \mathsf{O_1}\phi$ iff $\exists \Delta \subseteq_{Fin} \Phi(\varnothing \neq [\wedge\Delta]^M \subseteq [\phi]^M)$;
$M, w \models \mathsf{O_U}\phi$ iff $\exists \Delta \subseteq_{Fin} \Phi(\varnothing \neq [\wedge\Delta]^M \subseteq [\phi]^M)$ and $M, w \models \neg\mathsf{O_1}\neg(\wedge\Delta))$.

As we can see, being unproblematically obligatory entails being obligatory.

Interpreting the two-level framework The connection between the two-level framework and the present one is the following. Let $M = (W, \Phi, V)$ be a two-level model and define $M^* = (W^*, \mathcal{R}, \mathcal{R}^+, \mathcal{S}^+, V^*)$ so that $W^* = W$, $\mathcal{R} = \{[\phi]^M : \phi \in \Phi\}$, $\mathcal{S}^+ = \mathcal{R}^+$ is the closure of $\mathcal{R}$ under consistent aggregation and conditioned addition; $V^* = V$.

Proposition 4.1 *$M, w \models \mathsf{O_0}\phi$ iff $M^*, w \models \mathsf{R_B}\phi$.*

Proof. $M, w \models \mathsf{O_0}\phi$ iff $\exists \phi_i \in \Phi([\phi]^M = [\phi_i]^M)$; iff $[\phi_i]^{M^*} \in \mathcal{R}$. □

Proposition 4.2 *$M, w \models \mathsf{O_1}\phi$ iff $M^*, w \models \mathsf{R_D}\phi$.*

Proof. $M, w \models \mathsf{O_1}\phi$
iff $\exists \Delta \subseteq_{Fin} \Phi([\wedge\Delta]^M \neq \varnothing$ and $[\wedge\Delta]^M \subseteq [\phi]^M)$
iff $\exists \mathcal{X} \subseteq \mathcal{R}(\bigcap \mathcal{X} \neq \varnothing$ and $\bigcap \mathcal{X} \subseteq [\phi]^{M^*})$, by def. $\mathcal{R}$
iff $\exists X \in \mathcal{R}^+(X \subseteq [\phi]^{M^*})$, by def. $\mathcal{R}^+$, since $\mathcal{X}$ is finite □

As a corollary we get that $M, w \models \neg\mathsf{O_1}\neg\phi$ iff $\forall X \subseteq \mathcal{R}^+(X \cap [\phi]^{M^*} \neq \varnothing)$.

Proposition 4.3 *$M, w \models \mathsf{O_U}\phi$ iff $M^*, w \models \mathsf{O_R}\phi$.*

Proof. $M, w \models \mathsf{O_U}\phi$
iff $\exists \Delta \subseteq_{Fin} \Phi([\wedge\Delta]^M \neq \varnothing$, $[\wedge\Delta]^M \subseteq [\phi]^M$ and $M, w \models \neg\mathsf{O_1}\neg(\wedge\Delta))$
iff $\exists \Delta \subseteq_{Fin} \Phi([\wedge\Delta]^M \neq \varnothing$, $[\wedge\Delta]^M \subseteq [\phi]^M$, $\forall X \subseteq \mathcal{R}^+(X \cap [\wedge\Delta]^{M^*} \neq \varnothing))$
iff $\exists \mathcal{Y} \subseteq \mathcal{R}(\bigcap \mathcal{Y} \neq \varnothing$ and $\bigcap \mathcal{Y} \subseteq [\phi]^M$ and $\forall X \subseteq \mathcal{R}^+(X \cap \bigcap \mathcal{Y} \neq \varnothing))$
iff $\exists Y \in \mathcal{R}^+(Y \subseteq [\phi]^M$ and $\forall X \subseteq \mathcal{R}^+(X \cap Y \neq \varnothing))$, since $\mathcal{X}$ is finite
iff $\exists Y \in \mathcal{R}^+ \forall X \in \mathcal{R}^+(X \cap Y \neq \varnothing$ and $Y \subseteq [\phi]^M)$, by logic
iff $\forall X \in \mathcal{R}^+(X \cap r([\phi]^M))$ by the definition of r □

As a consequence, *obligations turn out to coincide with derivative reasons, while unproblematic obligations coincide with reason-based obligations.* Hence, McNamara's two-level framework can be interpreted as the fragment of *RDL* dealing with derivative reasons and reason-based obligations.

[15] This is a semantic version of the truth conditions proposed in [15, 148-150]. See [15, sec.2] for a detailed presentation of the system and a justification of the truth conditions.

4.2 Horty's default system

Let $(\mathcal{L}, \vdash)$ be a system of classical propositional logic.[16] A *default rule* r is a pair $(\mathbf{a}[r], \mathbf{c}[r])$, where $\mathbf{a}[r], \mathbf{c}[r] \in \mathcal{L}$ are the antecedent and the consequent of r. In terms of reasons, r states that $\mathbf{a}[r]$ is a reason to do $\mathbf{c}[r]$. A *default theory* is a triple $(\mathcal{W}, \mathcal{D}, <)$ where $\varnothing \neq \mathcal{W} \subseteq \mathcal{L}$ is a consistent set of background information, $\mathcal{D} \neq \varnothing$ a set of default rules and $<$ an irreflexive and transitive relation on $\mathcal{D}$. A *scenario* is a set of rules. If S is a scenario, then $\mathbf{a}[S] = \{\mathbf{a}[r] : r \in S\}$ and $\mathbf{c}[S] = \{\mathbf{c}[r] : r \in S\}$. If S is a scenario and $r \in \mathcal{D}$, we say that

(i) $Tr[S] = \{r \in \mathcal{D} : \mathcal{W}, \mathbf{c}[S] \vdash \mathbf{a}[r]\}$
is the set of rules that are *triggered* in S.

(ii) $Cr[S] = \{r \in \mathcal{D} : \mathcal{W}, \mathbf{c}[S] \vdash \neg\mathbf{c}[r]\}$
is the set of rules that are *conflicted* in S.

(iii) $Dr[S] = \{r \in \mathcal{D} : \exists d \in Tr[S](r < d \text{ and } r \in Cr[d])\}$
is the set of rules that are *defeated* in S.

If S is a scenario, then S is *consistent* iff $S = S - Cr[S]$ and S is *proper* iff $S = Tr[S] - Cr[S] - Dr[S]$. Thus, a scenario is consistent provided that it is conflict free and it is proper provided that it contains all and only the triggered rules that are not conflicted or defeated in it.

Remark 4.4 *Say that a default theory is* basic *when* $\forall\phi(\mathcal{W}, \mathbf{c}[\mathcal{D}] \Vdash \phi \Leftrightarrow \mathcal{W} \Vdash \phi)$. *Then,* $Tr[S] = Tr[\mathcal{D}]$ *and* $Dr[S] = Dr[\mathcal{D}]$ *for all* $S \subseteq \mathcal{D}$. *Thus, there is a unique set* $Tr[\mathcal{D}] - Dr[\mathcal{D}]$ *of undefeated rules and* S *is proper iff* S *is consistent, that is if and only if* $S = S - Cr[S] \subseteq Tr[\mathcal{D}] - Dr[\mathcal{D}]$.[17]

Let $\mathcal{D}^*$ be the union of the set of proper scenarios, $\mathbf{c}[\mathcal{D}^*]$ be the set of consequents of rules in $\mathcal{D}^*$, and $\mathcal{E}[\mathbf{c}[\mathcal{D}^*]]$ be the set of maximal consistent subsets of $\mathbf{c}[\mathcal{D}^*]$. The elements of $\mathcal{E}[\mathbf{c}[\mathcal{D}^*]]$ are then the most inclusive *objectives* available to an agent given the reasons in $\mathcal{D}$ and the background information $\mathcal{W}$.[18]

Definition 4.5 *Let* $(\mathcal{W}, \mathcal{D}, <)$ *be a default theory. Then, we can define two operators,* S_h *and* O_h*, corresponding to derivative reason and obligation* a là *Horty, by introducing the following truth conditions.*

1. $(\mathcal{W}, \mathcal{D}, <) \models \mathsf{S}_h\phi$ iff $\Delta \Vdash \phi$ for some $\Delta \in \mathcal{E}[\mathbf{c}[\mathcal{D}^*]]$.
2. $(\mathcal{W}, \mathcal{D}, <) \models \mathsf{O}_h\phi$ iff $\Delta \Vdash \phi$ for every $\Delta \in \mathcal{E}[\mathbf{c}[\mathcal{D}^*]]$.

Hence, there is a derivative reason to do ϕ iff ϕ is entailed by some objective and there is an obligation to do ϕ iff ϕ is entailed by every objective.

Interpreting the default framework To provide a representation of a default theory in the framework of RDL, I assume that the theory we want to represent contains a finite number of rules and a rule to the effect that agents have to take into account the background information, so that what is set-

[16] See [13,14]. In [16] a version of this system is used to model the connection between reasons and obligations. Here I will only consider fixed priority default versions.

[17] The system in [16] is essentially a basic default theory.

[18] Here it is assumed that $\mathcal{D}^* \neq \varnothing$. This entails that there is a proper scenario in $\mathcal{D}$.

tled given $\mathcal{W}$ is something that cannot be contrasted. A rule of this kind is $(\top, \wedge\{\mathcal{W}\})$ and requires that $\mathcal{W}$ be finite. So, let us say that a default theory $(\mathcal{W}, \mathcal{D}, <)$ is *suitable* when $\mathcal{W}$ and $\mathcal{D}$ are *finite* and $(\top, \wedge\{\mathcal{W}\})$ is contained in every proper scenario. Let $[\phi]$ be the set of maximal RDL-consistent sets containing ϕ and $M = (W, \mathcal{R}, \mathcal{R}^+, \mathcal{S}^+, V)$ be such that

W is the set of maximal RDL-consistent sets including $\mathcal{W}$;
$\mathcal{R} = \{[\phi] : \phi \in \mathbf{c}[\mathcal{D}]\}$;
$\mathcal{R}^+$ is the closure of $\mathcal{R}$ under combinations of reasons;
$\mathcal{S}^+$ is the closure of $\{[\phi] : \phi \in \mathbf{c}[\mathcal{D}^*]\}$ under combinations of reasons.

$W \neq \varnothing$, since $\mathcal{W}$ is consistent, and is to be identified with the set of states that are possible in light of what is settled given the background information. It is evident that $M = (W, \mathcal{R}, \mathcal{R}^+, \mathcal{S}^+, V)$ is a model for RDL. Now, let $(\mathcal{W}, \mathcal{D}, <)$ be *suitable.*

Proposition 4.6 *If no modality is in ϕ, then $(\mathcal{W}, \mathcal{D}, <) \models \mathsf{S}_h\phi$ iff $M \models \Diamond\mathsf{S}\phi$.*

Proof. $M, w \models \Diamond\mathsf{S}\phi$ iff $\exists X \in \mathcal{S}^+(X \subseteq [\phi]^M)$
iff $\exists\theta_1, ..., \theta_N \in \mathbf{c}[\mathcal{D}^*](\varnothing \neq [\theta_1]^M \cap ... \cap [\theta_N]^M \subseteq [\phi]^M)$
iff $[\Delta]^M \subseteq [\phi]^M$ for some $\Delta \in \mathcal{E}[\mathbf{c}[\mathcal{D}^*]]$, since $\mathcal{D}$ is finite
iff $\mathcal{W}, \Delta \vdash_{RDL} \phi$ for some $\Delta \in \mathcal{E}[\mathbf{c}[\mathcal{D}^*]]$, by the definition of W
iff $\Delta \vdash_{RDL} \phi$ for some $\Delta \in \mathcal{E}[\mathbf{c}[\mathcal{D}^*]]$, since $\wedge\{\mathcal{W}\}$ is in every Δ
iff $\Delta \vdash \phi$ for some $\Delta \in \mathcal{E}[\mathbf{c}[\mathcal{D}^*]]$, since $\phi \in \mathcal{L}$
iff $(\mathcal{W}, \mathcal{D}, <) \models \mathsf{S}_h\phi$ □

Proposition 4.7 *If no modality is in ϕ, then $(\mathcal{W}, \mathcal{D}, <) \models \mathsf{O}_h\phi$ iff $M \models \mathsf{O}_\mathsf{S}\phi$.*

Proof. $M, w \models \mathsf{O}_\mathsf{S}\phi$ iff $\forall X \in \mathcal{S}^+\exists Y \in \mathcal{S}^+(Y \subseteq X \cap [\phi]^M)$; by the definition of $\mathcal{S}^+$ and the finiteness of $\mathcal{D}$, this is equivalent to $[\Delta]^M \subseteq [\phi]^M$ for all $\Delta \in \mathcal{E}[\mathbf{c}[\mathcal{D}^*]]$; the rest of the proof is then similar to the previous one. □

Models for RDL can be regarded as semantic generalizations of default theories.[19] To be sure, the notions of being conflicted and being defeated are definable in terms of the consequents of the rules in $\mathcal{D}$: two rules conflict when their consequents cannot be realized together, while the ordering on $\mathcal{D}$ can be based on an ordering on consequents, which are the items that are assessed as to their practical weight. In RDL we abstract both from the structure of the rules and from the specific procedure used to identify what scenarios are proper. This is a benefit in terms of flexibility, but a cost in terms of transparency, since the work done by the ordering relation is completely incorporated in the implicit choice function that allows us to pick out $\mathcal{S}^+$ from $\mathcal{R}^+$.

5 Conclusion

Consider the following core principles of standard deontic logic.

[19] Since reasons derive from triggered rules, we are also able to interpret conditional rules like $(\mathbf{a}[r], \mathbf{c}[r])$ in terms of conjunctive reasons $W - (X \cap -Y)$, where X is the set of states where $\mathbf{a}[r]$ is realized and Y is the set of states where $\mathbf{c}[r]$ is realized.

1. $\Box(\phi \leftrightarrow \psi) \wedge \mathsf{O}\phi \to \mathsf{O}\psi$;
2. $\Box\phi \to \mathsf{O}\phi$;
3. $\mathsf{O}\phi \to \Diamond\phi$;
4. $\mathsf{O}\phi \to \neg\mathsf{O}\neg\phi$;
5. $\mathsf{O}\phi \wedge \mathsf{O}\psi \to \mathsf{O}(\phi \wedge \psi)$;
6. $\Box(\phi \to \psi) \wedge \mathsf{O}\phi \to \mathsf{O}\psi$;
7. $\mathsf{O}\phi \wedge \mathsf{O}\psi \wedge \Diamond(\phi \wedge \psi) \to \mathsf{O}(\phi \wedge \psi)$.

Systems including $\{3, 5\}$ or $\{4, 6\}$ do not allow for conflicts, while systems including $\{5, 6\}$ or $\{6, 7\}$ do not avoid explosions. So, in order to allow for conflicts and avoid explosions one principle in each of $\{3, 5\}$, $\{4, 6\}$, $\{5, 6\}$, $\{6, 7\}$ is to be discarded. In reason-based deontic logic, the picture is as follows.

	1.	2.	3.	4.	5.	6.	7.
R_B	✓	✓	✓				
R_C	✓	✓	✓				✓
R_D	✓	✓	✓			✓	
O_R and O_S	✓	✓	✓	✓	✓	✓	✓

O_R and O_S are conflict-free operators, given that they model kinds of all things considered obligation resulting from agent deliberation. In addition, since principles $4, 5, 6$ are invalid relative to R_B and R_C and $4, 5, 7$ are invalid relative to R_D, all of R_B, R_C, R_D are conflict tolerant operators that enable us to avoid explosions (theorem 3.13) and to construct arguments based on IR and DR (theorem 3.15). Thus, RDL provides us with an intuitive way to integrate reasons and obligations in a coherent system. This integration is based on the notion of combined reason, which is worth considering both from a philosophical point of view (it allows us to focus on two basic operations characterizing practical reasoning, namely consistent aggregation and conditioned addition) and from a logical point of view (it allows us to connect the logic of reasons with the logic of obligation and to develop a complete system of reason-based deontic logic). Also, RDL gives us a principled framework for addressing issues concerning deontic principles. Indeed, RDL was not obtained by constraining some deontic principles in order to avoid counter-intuitive conclusions. [20] To the contrary, we have first introduced elementary principles on how to combine reasons, and then demonstrated how solutions to pressing deontic problems follow from these principles, given suitable definitions of the deontic operators. Finally, RDL is connected with evidence-based systems of epistemic logic, thus providing us with a helpful basis for developing a unified account of reasons in deontic and epistemic contexts.

[20] This complaint is expressed in [12, p. 311]: "The kind of neighborhood semantics described above, while valuable for establishing results about the logics, such as determining what is derivable from what within the systems, do not yield much illumination into the concepts being formalized. The conditions on the neighborhoods that validate the various principles merely mimic, at the level of propositions, the principles being validated".

References

[1] Audi, R., *The Architecture of Reason: The Structure and Substance of Rationality*, Oxford University Press, Oxford, 2001.

[2] Audi, R., *Practical Reasoning and Ethical Decision*, Routledge, London, 2006.

[3] Baltag, A. and S. Smets, *A qualitative theory of dynamic interactive belief revision*, in: van der Hoek, W. and M. Wooldridge, editors, *Texts in Logic and Games 3*, Amsterdam University Press, Amsterdam, 2008, pp. 11–58.

[4] Baltag, A., N. Bezhanishvili, A. Özgün and S. Smets, *Justified belief and the topology of evidence*, WoLLIC, Springer, Berlin, 2016, pp. 83–103.

[5] Baltag, A., N. Bezhanishvili, A. Özgün and S. Smets, *A topological approach to full belief*, Journal of Philosophical Logic, 48 (2019), pp. 205–244.

[6] Brink, D., *Moral conflict and its structure*, Philosophical Review, 103 (1994), pp. 215–247.

[7] Broome, J., *Rationality Through Reasoning*, Blackwell, Oxford, 2013.

[8] Chellas, B., *Modal logic. An Introduction*, Cambridge University Press, Cambridge, 1980.

[9] Fumerton, R., *Epistemology*, Blackwell, Oxford, 2006.

[10] Goble, L., *A Logic for Deontic Dilemmas*, Journal of Applied Logic, 3 (2005), pp. 461–483.

[11] Goble, L. *Normative conflicts and the logic of 'ought'*, Nous, 43 (2009), pp. 450–489.

[12] Goble, L., *Prima facie norms, normative conflicts, and dilemmas*, in: D. Gabbay, J. Horty, X. Parent, R. van der Meyden and L. van der Torre, editors, *Handbook of deontic logic and normative systems*, College Publications, Milton Keynes, 2013, pp. 241–352.

[13] Horty, J., *Reasoning with moral conflicts*, Nous, 37 (2003), pp. 557–605.

[14] Horty, J., *Reasons as defaults*, Oxford University Press, Oxford, 2012.

[15] McNamara, P., *Agential obligation as non-agential personal obligation plus agency*, Journal of Applied Logic, 2 (2004), pp. 117–152.

[16] Nair, S., *Conflicting reasons, unconflicting 'ought's*, Philosophical Studies, 173 (2016), pp. 629–663.

[17] Nair, S. and J. Horty, *The logic of reasons*, in: D. Star, editor, *The Oxford Handbook of Reasons and Normativity*, Oxford University Press, Oxford, 2018, pp. 67–85.

[18] Pacuit, E., *Neighborhood Semantics for Modal Logic*. Springer, Berlin, 2017.

[19] Parfit, D., *On what matters*, Volume 1, Oxford University Press, Oxford, 2011.

[20] Raz, J., *Practical reasoning and norms*, Oxford University Press, Oxford, 1999.

[21] Raz, J., *Engaging Reason: On the Theory of Value and Action*, Oxford University Press, Oxford, 2002.

[22] Scanlon, T. M., *What we owe to each other*, Harvard University Press, Cambridge, 1998.

[23] Sen, A., *Choice Functions and Revealed Preference*, The Review of Economic Studies, 38 (1971), pp. 307–317.

[24] Sen, A., *Collective Choice and Social Welfare. An Expanded Edition*, Harvard University Press, Cambridge, 2017.

[25] Star, D., editor, *The Oxford Handbook of Reasons and Normativity*, Oxford University Press, Oxford, 2016.

[26] van Benthem, J. and E. Pacuit, *Dynamic logics of evidence-based beliefs*, Studia Logica, 99 (2011), pp. 61–92.

[27] van Benthem, J., D. Fernndez-Duque and E. Pacuit, *Evidence Logic: A New Look at Neighborhood Structures*, Advances in modal logic, 9 (2012), pp. 97–118.

[28] van Benthem, J., D. Fernandez-Duque and E. Pacuit, *Evidence and plausibility in neighborhood structures*, Annals of Pure and Applied Logic, 165 (2014), pp. 106–133.

[29] van der Torre, L., and Y. H. Tan, *Two phase deontic logic*, Logique et Analyse, 171–172 (2000), pp. 411–456.

[30] van Fraassen, B., *Values and the heart's command*, Journal of Philosophy, 70 (1973), pp. 5–19.